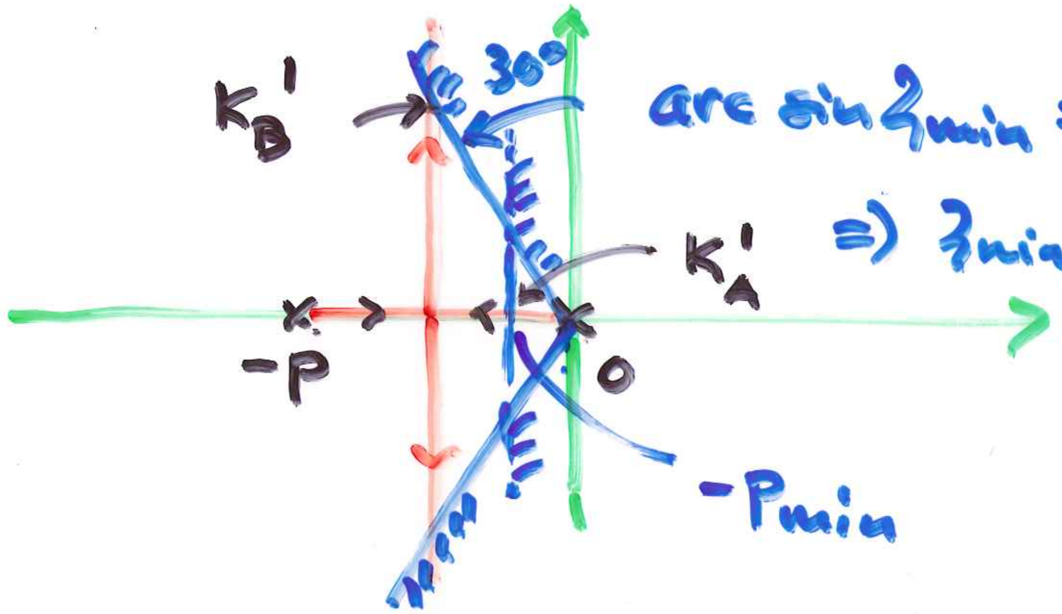
ex (uso di reti correttrici)

$$P(s) = \frac{1}{s(s+p)} \quad p>0 \qquad C(s) = \{ k' \ldots$$



arc sin $\zeta_{min} = 30°$

$\Rightarrow \zeta_{min} = 0.5$

specifiche (oltre la stab. asint.) sul r. perm. e sul transitorio (area entro i tratteggi $\leftrightarrow p_{min}, \zeta_{min}$)

$$e_1 = \frac{K_d^2}{K_G} = \frac{1}{k' \cdot K_p} = \frac{p}{k'} \leq e_{1,max}$$ su rampa unitaria

$$\boxed{k' \geq \frac{p}{e_{1,max}}}$$

$$0 < k'_A \leq k' \leq k'_B$$

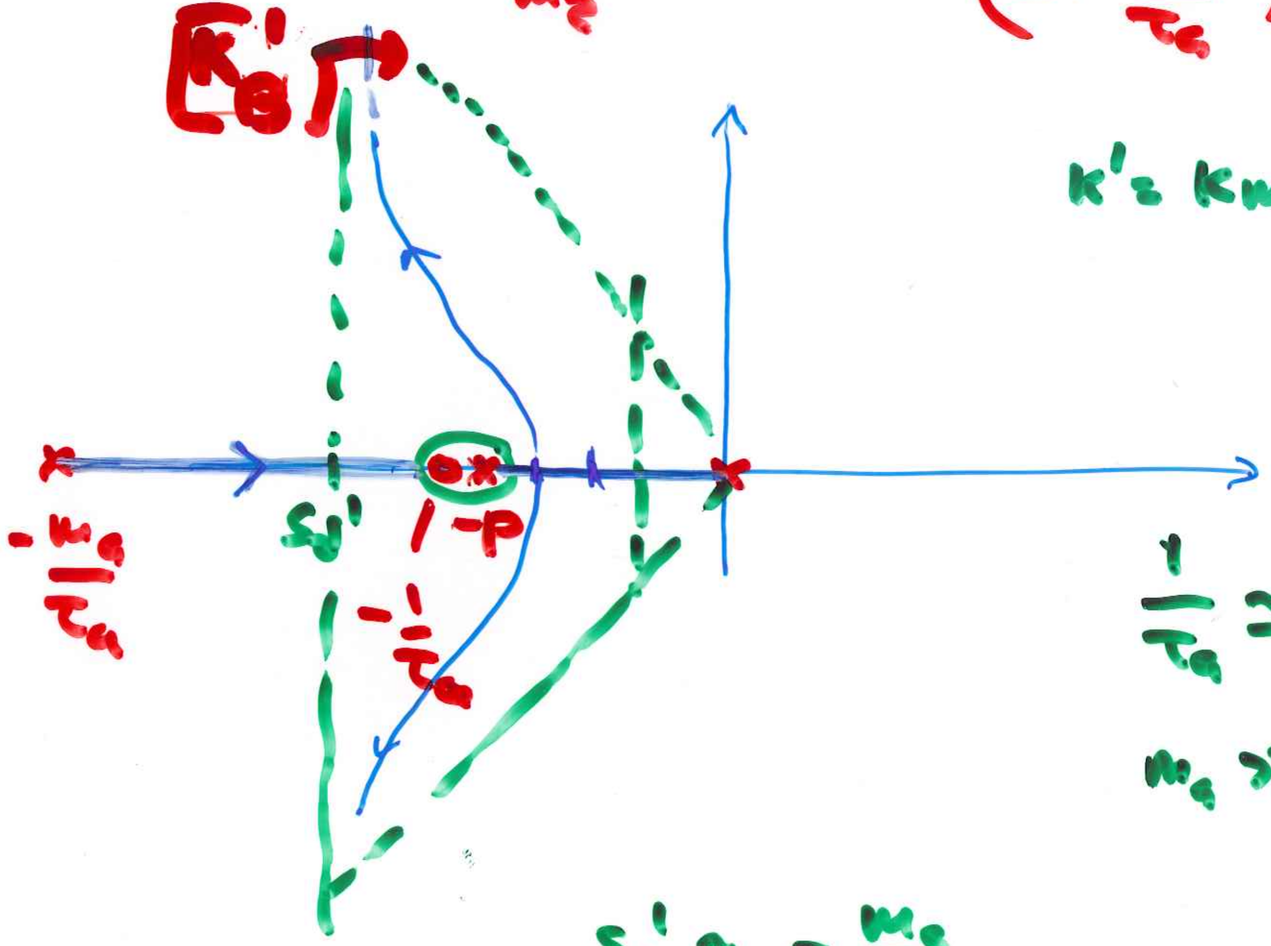
$0 < p_{min}(p - p_{min}) \qquad p \cdot p = p^2$

$\Rightarrow$ se $\frac{p}{e_{1,max}} \leq p^2$ allora k' va bene ... else ?

opt. 1) rete anticipatrice

$$C(s) = K\, R_2(s)$$

$$R_2(s) = \frac{1+\tau_a s}{1+\frac{\tau_a}{m_a} s} = m_a \frac{\left(s+\frac{1}{\tau_a}\right)}{\left(s+\frac{m_a}{\tau_a}\right)}$$



$K' = K m_a$

$\frac{1}{\tau_a} \simeq p$

$m_a \gg 1$

$$s_0' \simeq -\frac{m_a}{2\tau_a}$$

$$K_B' \simeq \left(\frac{m_a}{\tau_a}\right)^2 \simeq m_a^2 p^2 = m_a^2 K_B$$

l'intervallo utile per il guadagno $\underline{K}$ si espande di un fattore $\underline{m_a}$

opz. 2) rete attenuatrice

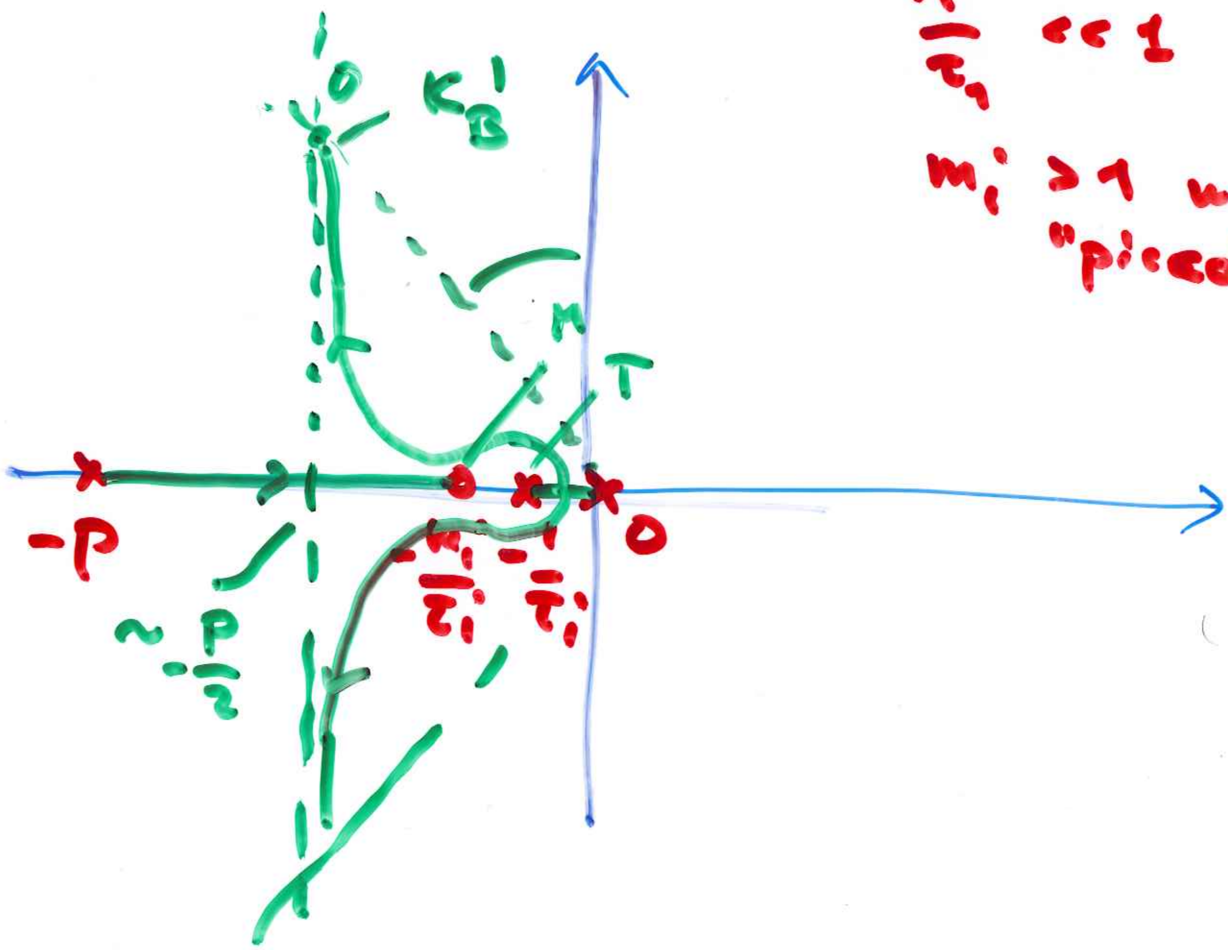
$$C(s) = K \, R_i(s)$$

$$R_i(s) = \frac{1 + \frac{\tau_i}{m_i} s}{1 + \tau_i s} = \frac{1}{m_i} \cdot \frac{s + \frac{m_i}{\tau_i}}{s + \frac{1}{\tau_i}} \qquad m_i > 1$$

$$K' = K \cdot \frac{1}{m_i}$$

$\frac{1}{\tau_1} \ll 1$

$m_i > 1$ ma "piccolo"



$$|K'_B| \cong \frac{P^2 \cdot |\vec{TO}|}{|\vec{MO}|} \simeq P^2$$

l'intervallo ammissibile per K' si amplifica di un fattore m_i

SINTESI PER SISTEMI A FASE MINIMA

- tutti gli (eventuali) zeri a parte reale < 0
- poli: stabili e/o instabili ad anello aperto

a) $n-m=1$ $\quad C(s)=K$

per $K>0$ suff. grande $\Rightarrow$ A.S.

b) $n-m=2$

b1) $$s_0 = \frac{\sum p_i - \sum z_j}{2} < 0$$

$C(s)=K$ per $K>0$ suff. grande $\Rightarrow$ A.S.

b2) $s_0 \geq 0$ $\quad C(s) = K\dfrac{s+z}{s+p}$ $\quad p>z>0$

$$s_0' = s_0 - \frac{p-z}{2} < 0$$

per $K>0$ suff. grande $\Rightarrow$ A.S.

c) $n-m > 2$

• $(s+z_1)(s+z_2)\dots(s+z_{n-m-2})$

(tutti a fase minima) $z_i > 0$

•• $\dfrac{s+z}{s+p} \Rightarrow s_0 < 0$

••• $K > 0$ e suff. grande

$$C(s) = \frac{K\,(s+z_1)(s+z_2)\dots(s+z_{n-m-2})(s+z)}{(s+p)\left[\prod(1+\tau_i s)\right]}$$

•••• per la realizzabilità del controllore

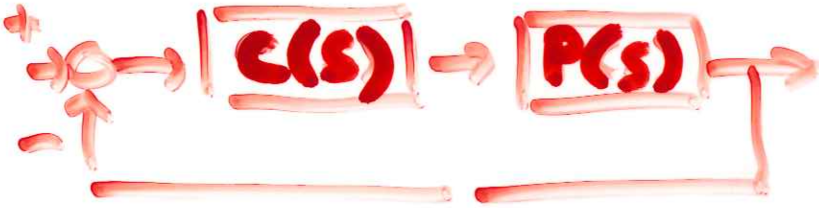
con $\tau_i > 0$ e suff. PICCOLO!

Thes

$W(s) = \dfrac{F(s)}{1+F(s)}$ è ASINT. STABILE

per τ suff piccolo ($\tau > 0$) allora $W'(s)$ è ASINT. STABILE

ex. di SINTESI

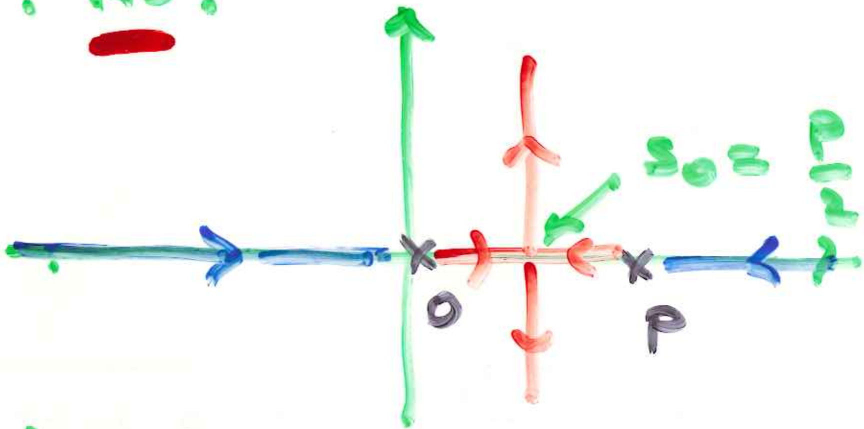


$P(s) = \frac{1}{s(s-p)}$

$p > 0$

Q: $C(s)$ stabilizzante? di ordine minimo?

$C(s) = K'$? NO!

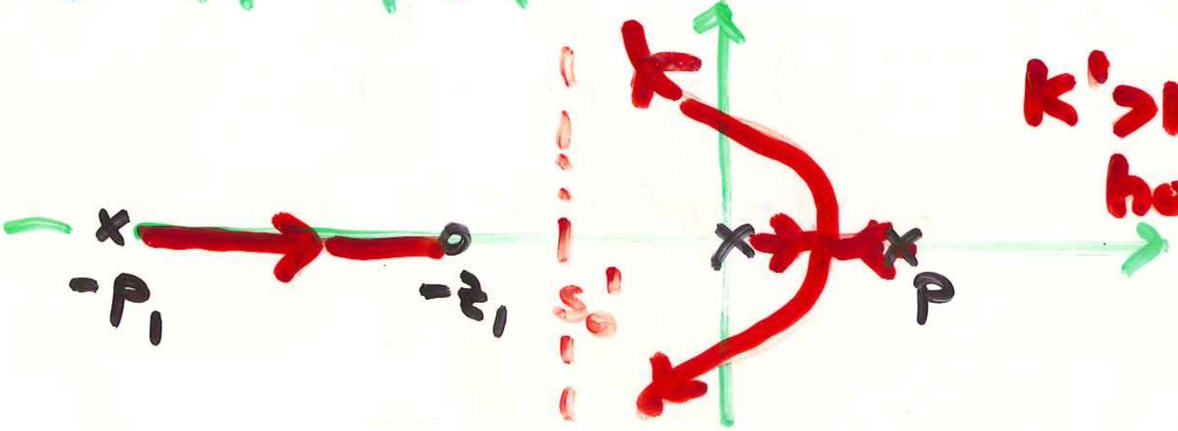


$C(s) = K' \frac{s+z_1}{s+p_1} \longrightarrow$ zero in $-z_1$, polo in $-p_1$

Hint: $z_1 > 0$, $p_1 > z_1 > 0$

$$s_0' = \frac{p - p_1 + z_1}{2} = s_0 - \underbrace{\frac{p_1 - z_1}{2}}_{>0} < 0 \;!!$$

$\Rightarrow \; p_1 - z_1 > p$



$K' > K'_{min} > 0$ ho stab. as.

(con Routh)

Routh

$$f(s,k') = s^3 + (p_1 - p)s^2 + (k' - pp_1)s + k'z_1$$

s^3	1	$k' - pp_1$
s^2	$p_1 - p$	$k'z_1$
s^1	$\dfrac{(k' - pp_1)(p_1 - p) - k'z_1}{p_1 - p}$	
s^0	$k'z_1$	

$$\dfrac{(k' - pp_1)(p_1 - p) - k'z_1}{p_1 - p} > 0 \Rightarrow k' > \dfrac{\overset{>0}{pp_1(p_1-p)}}{\underset{>0}{p_1 - z_1 - p}}$$

Commenti generali

i) $$C'(s) = \frac{K_c}{s}$$ (per il regime permanente)

$$F'(s) = C'(s)\,P(s)$$ $n-m = 3$

$$C''(s) = \frac{k\,(s+z_1)(s+z)}{(1+\tau_1 s)\,(s+p)}$$ (per il transitorio $\simeq$ asintotica stabilità)

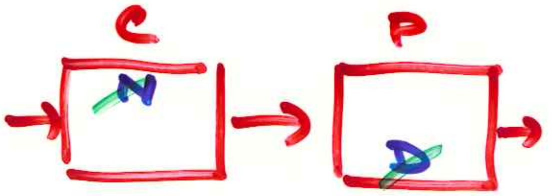
$\tau_1 > 0$
suff. piccolo

$$C(s) = C'(s)\,C''(s) = \frac{K_c\,k\,(s+z_1)(s+z)}{\underbrace{s}\,(s+p)\,\cancel{(1+\tau_1 s)}}$$

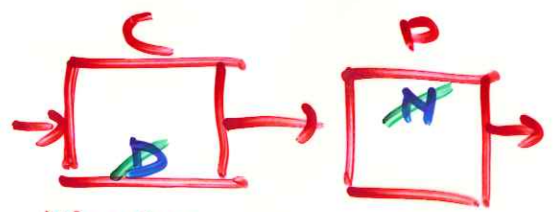
!!

ii) "se possibile", procedere per CANCELLAZIONE

- zeri a fase minima del controllore con poli (asint. stabili) del processo
- poli asintot. stabili del controllore con zeri (a fase minima) del processo



IRRAGGIUNGIBILI



INOSSERVABILI

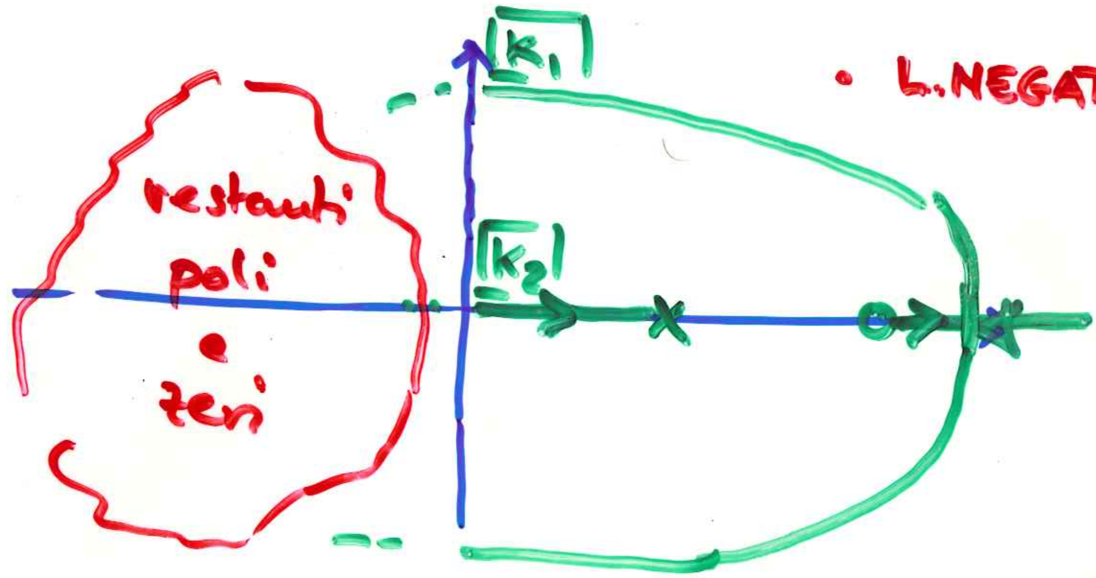
SINTESI PER SISTEMI A FASE NON MINIMA

• almeno uno zero è a parte reale positiva

a) sistema stabile asintoticamente ad anello aperto ⇒ $|K|$ suff. piccolo ⇒ mantiene A.S.
($C(s) = K$)

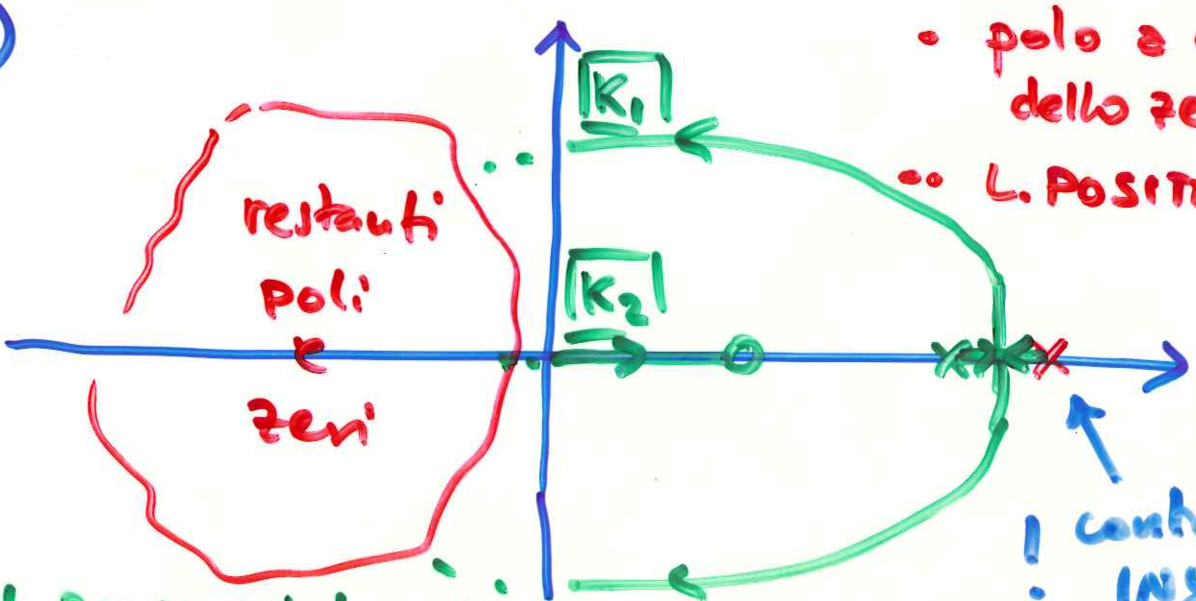
b) uno zero e un polo reali positivi

b1) • L. NEGATIVO



evtl. $K_1 < K' < K_2 < 0$

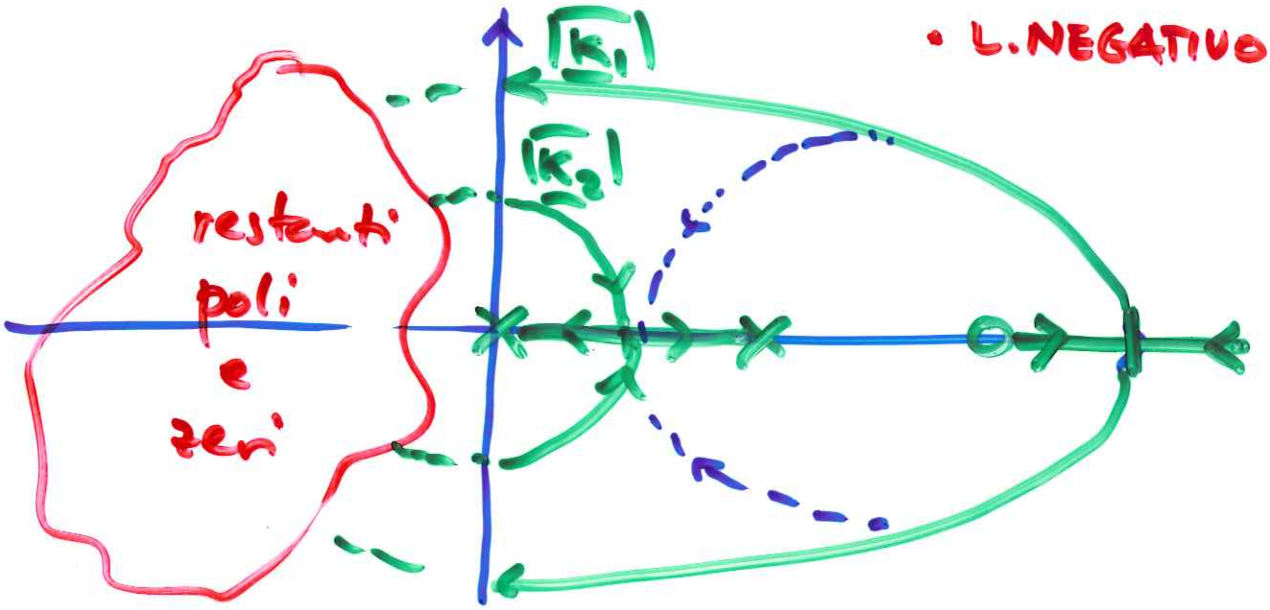
b2) • polo a dx dello zero
•• L. POSITIVO



! controllore INSTABILE

evtl. $0 < K_1 < K' < K_2$

c) polo in s=0 , polo/zero reali positivi



eventl. $k_1 < k' < k_2 < 0$

d) ELSE ... metodo DIRETTO di SINTESI del CONTROLLORE

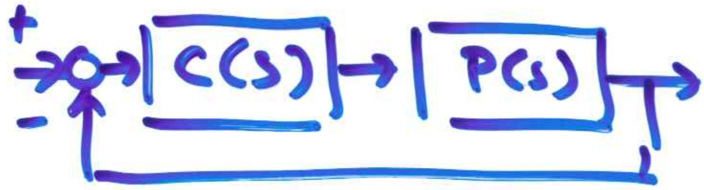
↓

impone i poli della $W(s)$ con un controllore generale ~~~~ di dimensione suff. grande

(principio di BEZOUT)

Assegnazione dei poli di $W(s)$ - SINTESI DIRETTA -

$$P(s) = \frac{N_p(s)}{D_p(s)} \qquad C(s) = \frac{N_c(s)}{D_c(s)}$$



$$D_p(s) = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0$$

$$N_p(s) = b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0$$

$$(m \leq n)$$

$$\left.\begin{aligned} D_c(s) &= s^r + c_{r-1}s^{r-1} + \ldots + c_1 s + c_0 \\ N_c(s) &= d_r s^r + d_{r-1}s^{r-1} + \ldots + d_1 s + d_0 \end{aligned}\right\} *$$

$\underline{2r+1}$ coefficienti incogniti

$$W(s) = \frac{N_p(s)\,N_c(s)}{D_p(s)\,D_c(s) + N_p(s)\,N_c(s)} \quad \left(= \frac{F(s)}{1+F(s)}\right.$$

$F(s) = C(s)\,P(s)$)

$$= \frac{''}{D_{des}(s)}$$

grado $\underline{n+r}$ monico (n+r coefficienti dati)

$$\Rightarrow \boxed{r = n-1}$$

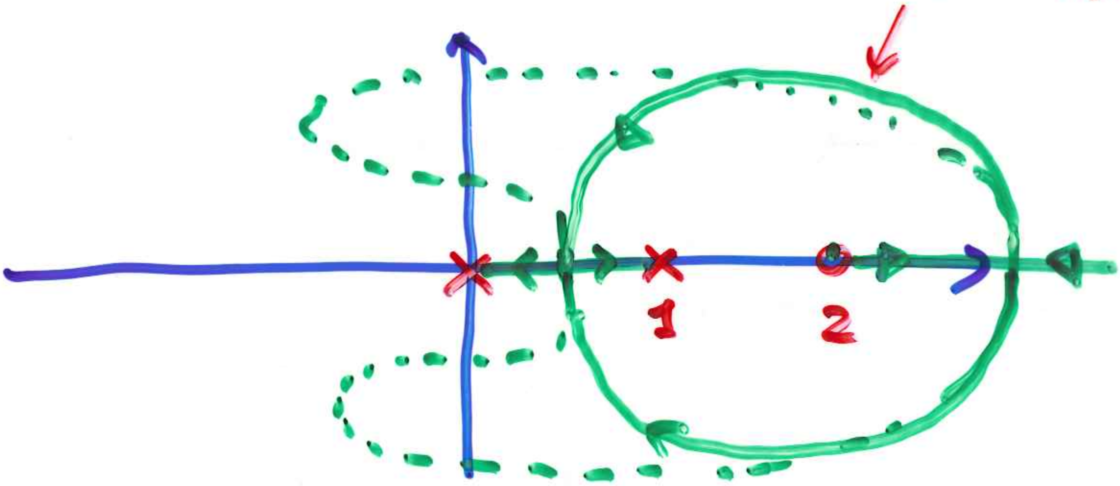
Ex. 1

$$P(s) = \frac{s-2}{s(s-1)} \qquad m=1 \qquad n=2$$

circonferenza centrata in $z=2$

$r = \sqrt{|2-p_1|\,|2-p_2|} = \sqrt{2}$

L.N.



$$C(s) = \frac{d_1 s + d_0}{s + c_0}$$

$$\text{den}\, W(s) = s(s-1)(s+c_0) + (s-2)(d_1 s + d_0)$$
$$= s^3 + (c_0 - 1 + d_1)s^2 + (d_0 - c_0 - 2d_1)s - 2d_0$$

$$D_{des}(s) = s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \quad \text{(Hurwitz)}$$

ad.es. tre poli in -1 $\rightleftarrows$ $(s+1)^3 = s^3 + 3s^2 + 3s + 1$

Routh

1	α_1
α_2	α_0
$(\alpha_1\alpha_2 - \alpha_0)/\alpha_2$	
α_0	

CN&S Hurwitz

$\alpha_2 > 0 \quad \alpha_0 > 0$

$\alpha_1 \alpha_2 > \alpha_0$

$$d_0 = -\frac{1}{2}$$

$$\begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ c_0 \end{bmatrix} = \begin{bmatrix} 3 - d_0 \\ 3+1 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 4 \end{bmatrix}$$

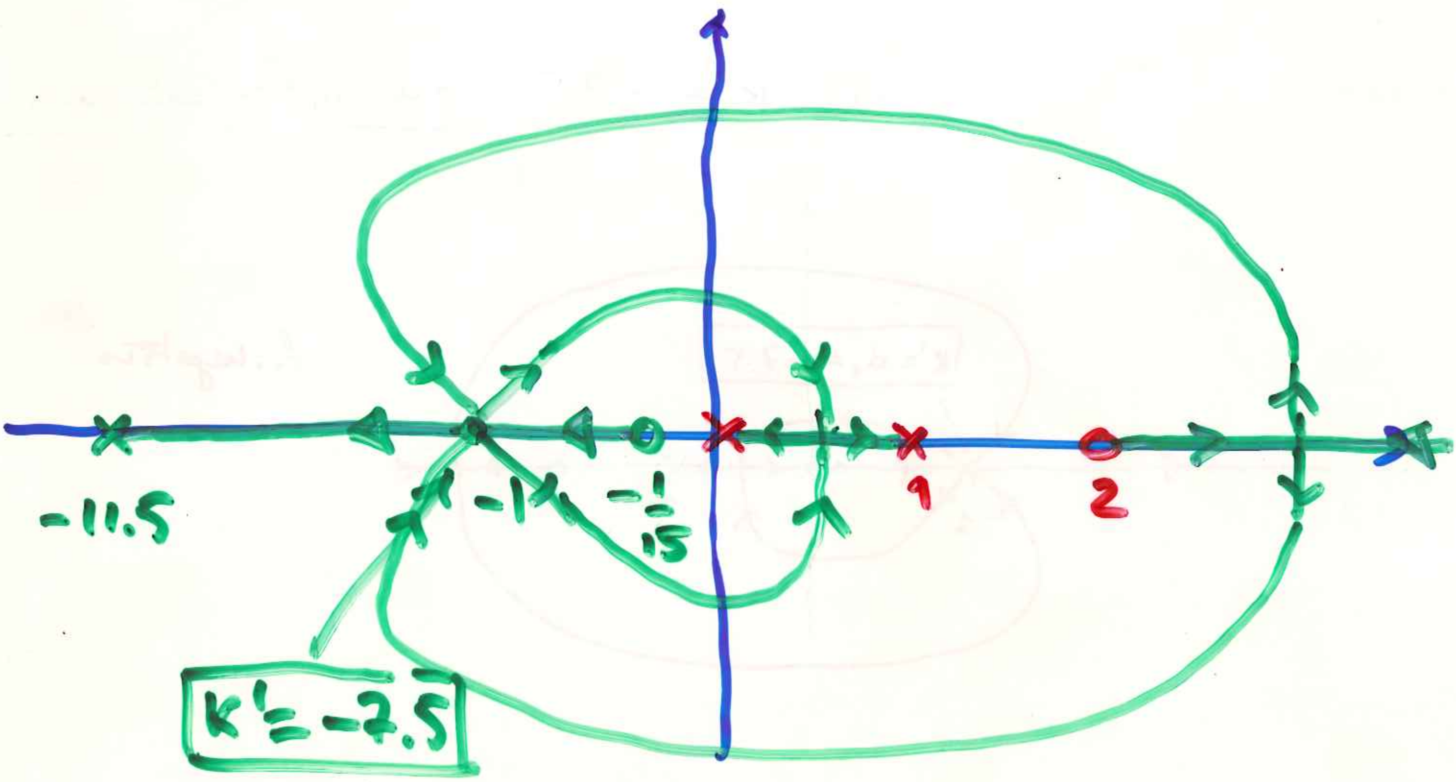
$$\Rightarrow \quad d_1 = -7.5 \qquad c_0 = 11.5$$

$$C(s) = \frac{-7.5\,s - 0.5}{s + 11.5} = \overset{K'}{\boxed{-7.5}}\ \frac{s + \frac{1}{15}}{s + 11.5}$$

un polo e uno zero reali negativi

$+ \ K' < 0$

L.R. <u>NEG</u>

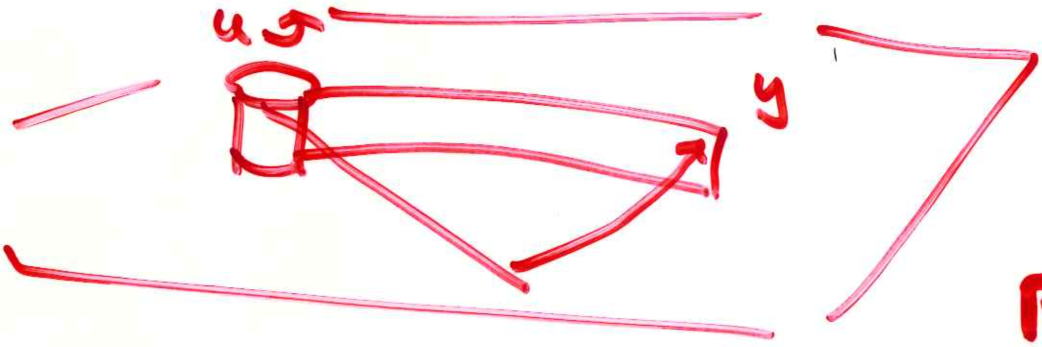


Ex. d'esame (#1, 16/9/97)

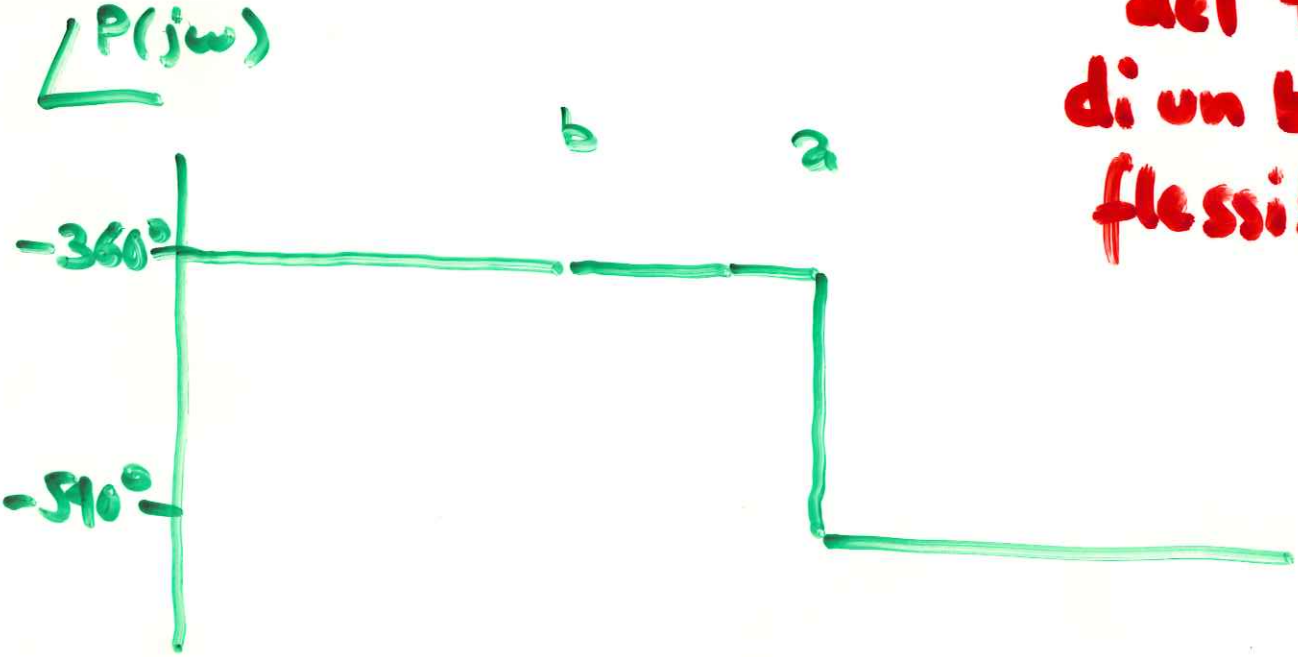
$$P(s) = \frac{s^2 - b^2}{s^2 (s^2 + a^2)} \qquad a > b \geq 1$$

i) tracciare in modo approx. il diagramma di Bode

ii) Progettare un controllore (reazione dall'uscita) in modo da avere A.S.

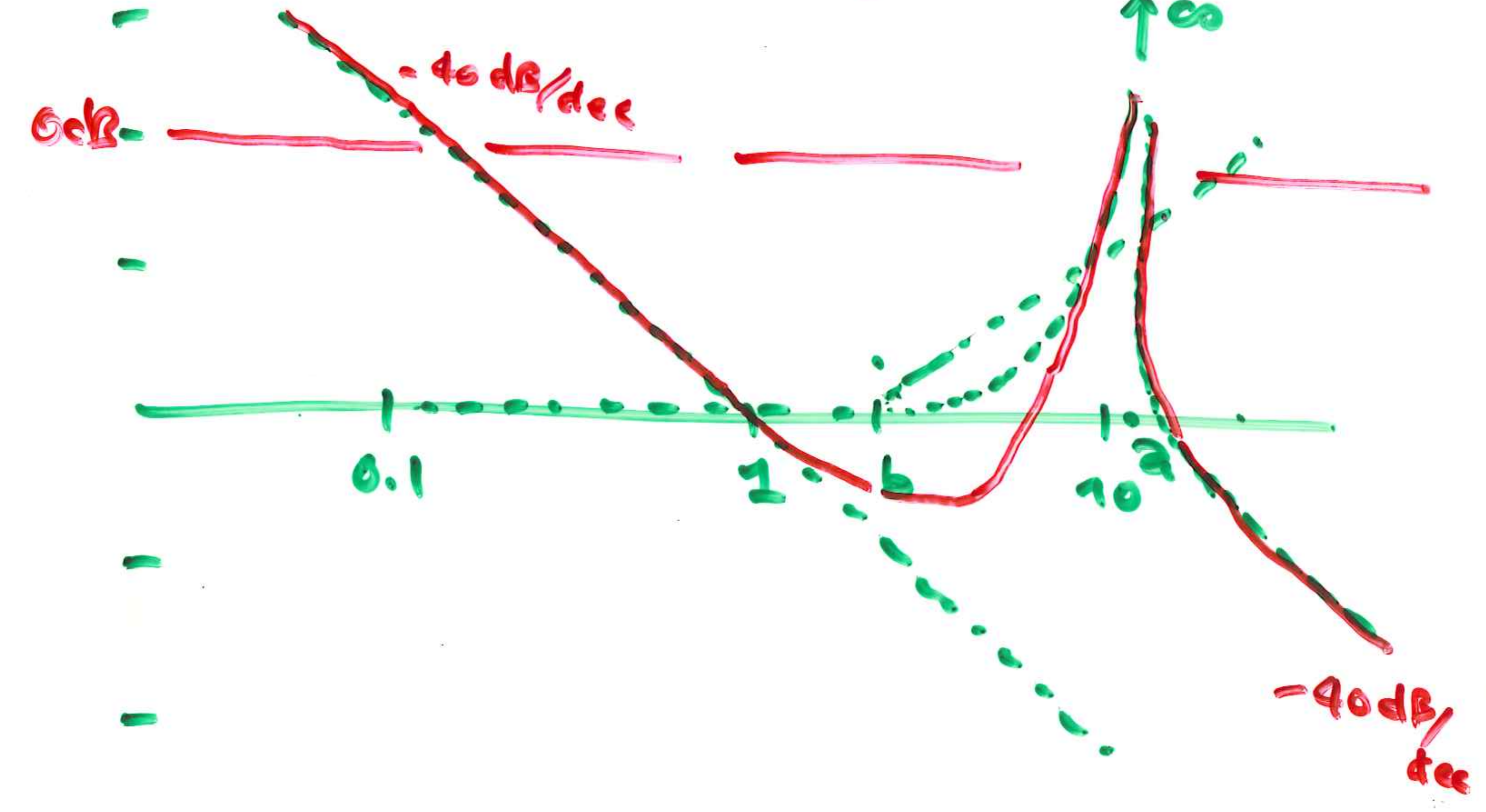


controllo di posizione del "tip" di un braccio flessibile



$|P(j\omega)|$

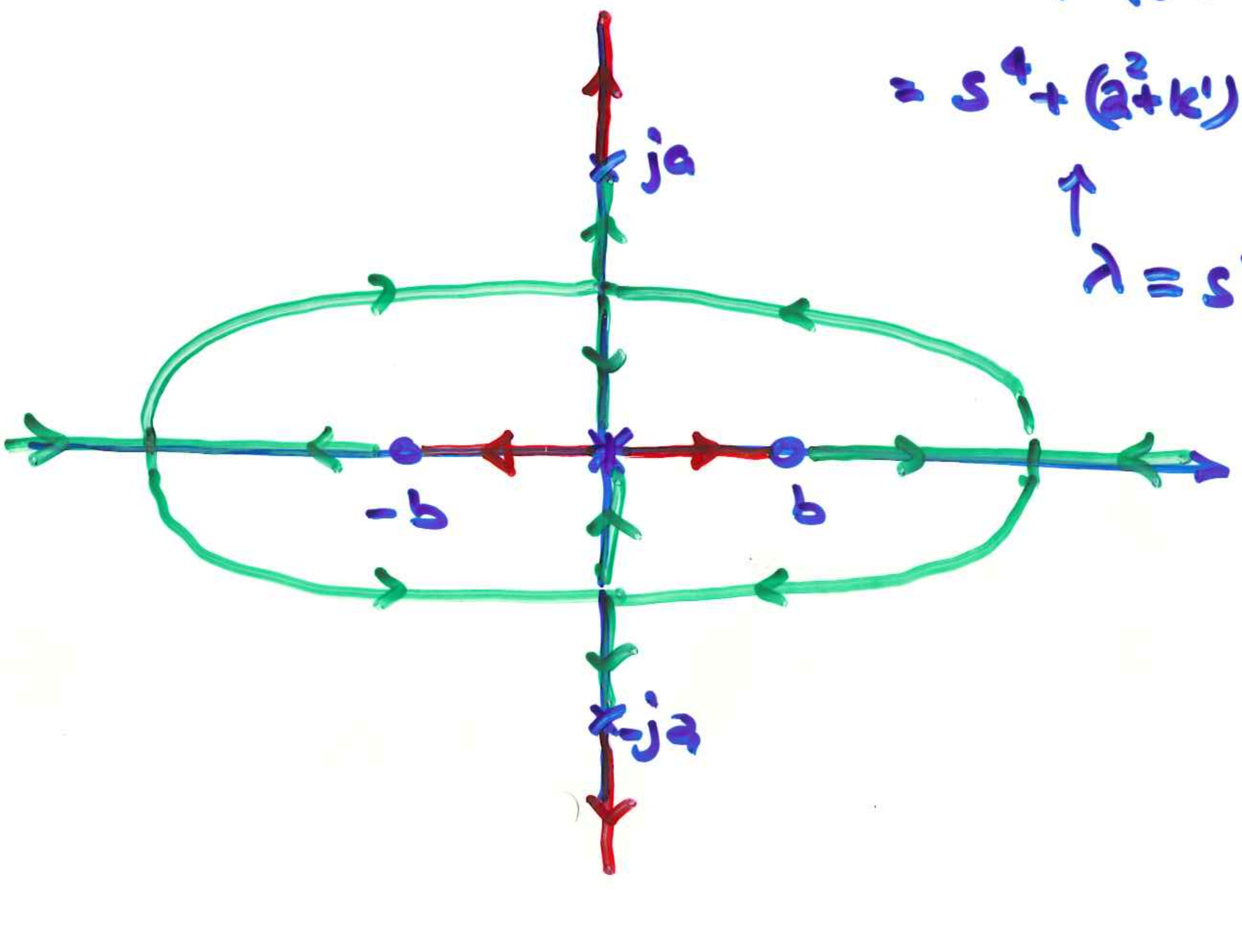
$|k_p| = \frac{b^2}{a^2} < 1$



L.Radic: LP LN

$$f(s,k') = s^2(s^2+a^2) + k'(s^2-b^2)$$
$$= s^4 + (a^2+k')s^2 - k'b^2$$

$$\uparrow$$
$$\lambda = s^2$$



metodo analitico

$$P(s) = \frac{s^2 - b^2}{s^2(s^2+a^2)} \qquad m = 2 \quad n = 4$$

Bezout $\Rightarrow$ $\exists\, C(s)$ di ordine $r = n-1 = 3$ che stabilizza asintoticamente $W(s)$ (in più posso assegnare arbitrariamente i $\boxed{7}$ poli di $W(s)$)

$$C(s) = \frac{d_3 s^3 + d_2 s^2 + d_1 s + d_0}{s^3 + c_2 s^2 + c_1 s + c_0} = \frac{n(s)}{d(s)}$$

$$D_{des}(s) = s^7 + \alpha_6 s^6 + \alpha_5 s^5 + \alpha_4 s^4 + \dots + \alpha_1 s + \alpha_0$$

di HURWITZ

$$\text{den}\, W(s) = d(s)\, s^2(s^2+a^2) + n(s)(s^2-b^2)$$

$$= s^7 + c_2 s^6 + (a^2 + c_1 + d_3)s^5 + (c_2 a^2 + c_0 + d_2)s^4 + (c_1 a^2 - b^2 d_3 + d_1)s^3 + (c_0 a^2 - b^2 d_2 + d_0)s^2 - b^2 d_1 s - b^2 d_0$$

$$d_0 = -\frac{\alpha_0}{b^2} \qquad d_1 = -\frac{\alpha_1}{b^2} \qquad c_2 = \alpha_6$$

rimangono 4 equazioni in 4 incognite

$$\begin{bmatrix} -b^2 & 0 & a^2 & 0 \\ 0 & -b^2 & 0 & a^2 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_2 \\ d_3 \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \alpha_2 - d_0 \\ \alpha_3 - d_1 \\ \alpha_4 - a^2 c_2 \\ \alpha_5 - a^2 \end{bmatrix}$$

.... calcoli per sostituzione

i) $$C(s)\Big|_{s=0} \underset{\uparrow}{=} \frac{d_0}{c_0} \; \underline{< 0} \; !!$$

$c_0 \neq 0 \Leftrightarrow$ nessun polo in $s=0$

ii) $C(s)$ per cancellazione

$$\downarrow$$

$$C(s) = \frac{d_3' s^3 + d_2' s^2 + d_1' s + d_0'}{(s+b)(s^2 + c_1' s + c_0')}$$